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in the work function of the surface, so that more of the electrons present may escape;[3] or it may be a light induced change in the chemical nature of the surface. In any case there must be ascribed to the light produced change a temperature coefficient of speed of response. Some clue to the nature of the phenomenon may perhaps be obtained from a study of the growth and decay curves. They are not simple exponential curves such as would represent the charging and discharging of a condenser. If the reciprocal of the square root of the decaying current increment is plotted against time, the plots are very nearly straight lines for all temperatures (law of phosphorescence decay). Attention may be called to the fact that the growth and decay of current represented in the curves in figure 3 exhibiting slow response at low temperatures and quick response at high, are quite similar to the growth and decay of current in selenium under illumination, which likewise show a great variation of speed with temperature. This analogy suggests that the cause of the light effect in the oxide coated filament may be closely related to that which gives selenium its photo-sensitive properties. It should also be noted that the change of speed of response with temperature would be expected if one assumed the effect to be due to a chemical reaction.

In view of the data presented it is believed that this photo-effect in thermionic filaments cannot safely be considered as evidence for a variation of the true photo-electric effect with temperature. We suggest the use of the term "photo-thermionic emission."

[1] T. W. Case, *Physic. Rev.*, **17**, 3, p. 398.

[2] E. Merritt, *Ibid.*, **17**, 4, p. 525.

[3] Koppius (*Ibid.*, Mar. 1921, p. 395) found with oxide coated filaments a decrease in the work function with increasing temperatures.

GROUP OF ISOMORPHISMS OF A TRANSITIVE SUBSTITUTION GROUP

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Let G represent any transitive substitution group of degree n. It is convenient to divide the possible automorphisms of G into three categories. First, those which can be obtained by transforming all the substitutions of G successively by each of its own substitutions. In this way the group of inner isomorphisms of G can be found and this group is known to be an invariant subgroup of the group of isomorphisms I of G. The second category consists of the automorphisms obtained by transforming G by substitutions on its own letters, which transform G into itself but are not found in G. It will be proved that these automorphisms together with the inner ones constitute all the possible automorphisms of G in which the subgroups composed separately of all the substitutions of G omitting a particular letter correspond to such subgroups.

The third category of automorphisms of G is composed of all those in which the subgroups composed of all the substitutions of G which omit a given letter correspond to subgroups of degree n. Such automorphisms are not always possible. In fact it may happen that I is simply isomorphic with the group of inner isomorphisms of G. In this case only the first category of automorphisms actually exists. A necessary and sufficient condition that the first category of automorphisms reduces to the identity is that G is abelian.

To prove the theorem noted at the end of the first paragraph we may consider any possible automorphism of G in which the subgroup G_1 composed of all the substitutions of G which omit a given letter a corresponds to itself while some of the substitutions of this subgroup do not necessarily correspond to themselves. In this automorphism every substitution of G which involves the letter a must correspond to such a substitution, and we shall suppose that the substitutions are represented in such a way that this letter appears first.

There are g/n, g being the order of G, substitutions of G in which the letter a is followed by any other letter b. If in one of the corresponding substitutions under the said automorphisms a is followed by d then this must be the case in each of the g/n substitutions which correspond to the substitutions beginning with the letters ab. If this were not the case as regards some two such corresponding substitutions then the inverses of the former of the two given corresponding substitutions into the latter would give two corresponding substitutions of which the former would omit the letter b while the latter would involve the letter d. This is impossible since the subgroup composed of all the substitutions which omit b must correspond to the subgroup composed of all the substitutions which omit d in the automorphism under consideration as a result of the fact that a substitution beginning with ab corresponds to one beginning with ad.

From the preceding paragraph it results that the second letters in all of the corresponding substitutions involving the letter a in the said automorphism determine a single substitution t on the letters of G. This substitution does not involve the letter a. It transforms G into a simply isomorphic group G' such that in the corresponding substitutions involving a the second letters are exactly the same as they are in the second group of the automorphism under consideration. If G were not identical with this second group, as a substitution group, it would be possible to establish a simple isomorphism between G and another substitution group G' such that every substitution involving a would correspond to such a substitution and that the second letter in all these corresponding substitutions would be the same in every case.

In this simple isomorphism every two corresponding substitutions would be identical. For, if S_1, S_1' are any two corresponding substitutions such that in S_1 the letter e is followed by the letter f, and if S_2, S_2' are two other corresponding substitutions in which the letter a is followed by e then S_1 is obtained by multiplying the inverse of S_2 into one of the g/n substitutions in which a is followed by f. Hence it results that in S_1' the letter e is also followed by the f. That is, S_1 and S_1' are identical. In this proof it was assumed that e is not a and that f is not a. If e were a then a would be replaced by f in S_1' by hypothesis. If f were a then in the inverse of S_1 the letter a would be replaced by e and hence in the inverse of S_1' the letter a would be replaced by e. Therefore the theorem again requires no further proof. Hence it has been established that G and G' are identical, and that the automorphism under consideration can be effected by transformed G by the substitution t which therefore transforms G into itself.

If the subgroup G_1 had corresponded in the given automorphism to a conjugate of G_1 but different from G_1 there would be a substitution in G which would transform G_1 into this conjugate. The inverse of this substitution into t would transform G into itself and effect the automorphism in question. It has therefore been proved that *every automorphism of G in which the subgroups composed of all the substitutions of G which omit a given letter correspond to such subgroups is effected by substitutions of the largest group on the letters of G which contains G invariantly.* As a particular case of this theorem we have the well known result that when G is regular it is transformed into all its possible automorphisms under its holomorph.

From the theorem proved in the preceding paragraph it is easy to deduce a method for finding the order of the group of isomorphisms of the transitive substitution group G. If G_1 is of degree $n - \alpha$ the largest group on the letters of G which contains G invariantly involves exactly α substitutions which are commutative with every substitution of G and hence this largest group transforms the substitutions of G according to a group whose order is the order of this largest group divided by α. This group of transformation is simply isomorphic with the quotient group of this largest group with respect to its invariant subgroup composed of the α substitutions which are commutative with every substitution of G. The order of the I of G is therefore equal to the order of this quotient group multiplied by the number of the different sets of conjugate subgroups of G which are such that each set involves a subgroup which corresponds to G_1 in some possible automorphism of G.

A necessary and sufficient condition that G contains more than one such set of conjugate subgroups is that it contains subgroups of degree n and of index n with respect to which it can be represented as a transitive substitution group which is identical with G. In particular, *if a given abstract group is simply isomorphic with only one group in a complete list of transitive groups of degree n then this transitive group has outer isomorphisms if it contains a subgroup of degree n and of index n which is non-invariant and does not involve any invariant subgroup of the entire group besides the identity.* From this special but useful theorem it follows directly that the symmetric group of degree 6 has outer isomorphisms. Various writers established this fact by somewhat laborious special methods.[1] It also follows directly from this theorem that the largest imprimitive groups on six letters which involve 2 and 3 systems of imprimitivity, respectively, have isomorphisms which cannot be obtained by transforming these groups by substitutions on their own letters.

While the group of inner isomorphisms is an invariant subgroup of I whenever it does not coincide with I it should not be inferred that the subgroup of I which corresponds to all the automorphisms of G which can be obtained by transforming G by substitutions on its own letters is always an invariant subgroup of I. In fact, this is not the case when G is the generalized dihedral group of order 16 involving the abelian group of type (2, 1) represented as a transitive group of degree 8. When G admits automorphisms in which G_1 corresponds to a subgroup of degree n the conjugates of G_1 are transformed under I according to an imprimitive substitution group. One of the systems of imprimitivity of this group is composed of the conjugates of G_1 under G.

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[1] Cf. O. Hölder, *Mathematische Annalen*, **46**, 1895 (345); W. Burnside, *Theory of Groups of Finite Order*, 1911, p. 209.

EINSTEIN STATIC FIELDS ADMITTING A GROUP G_2 OF CONTINUOUS TRANSFORMATIONS INTO THEMSELVES

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1. For static phenomena in the Einstein theory the linear element of the space-time continuum can be taken in the form $V^2dx_0^2-ds^2$, where

$$ds^2 = \Sigma\, a_{ik}\, dx_i\, dx_k\ (i,\ k = 1, 2, 3) \qquad (1)$$

is the linear element of the physical space S, and the functions V and a_{ik} are independent of x_0, the coördinate of time. In this paper we determine the functions V and a_{ik} satisfying the Einstein equations $B_{ik} = 0$ and such that the space S admits a continuous group G_2 of transformations into itself. Bianchi[1] has shown that any 3-space admitting such a group